THE USE OF DEMPSTER-SHAFER THEORY IN MEDICAL DECISION MAKING

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1. Introduction

The aim of this paper is twofold. First we view the basic notions of Shafer's theory of evidence (labeled also the Dempster-Shafer theory) in the context of more general mathematical theory, namely the theory, of random sets. Apart from the tutorial aspects this shows that the theory of evidence really a „mathematical" well-formed theory. Moreover such a point of view allows to simplify some of Shafer's proofs and makes them more universal. In this paper we omit mathematical details, however, referring an interested reader to papers [7] and [8]. Second, we try to show the flexibility of Shaferian apparatus in modelling human judgements. The emergence of expert systems makes critical the problem of managing uncertain information. Pure probabilistic methods and the certainty factor model used in MYCIN seem to be insufficient.

In medicine the precise determination of the degree of credibility of the diagnosis can indicate the possibly best method of the management of the patient. Parallel to the development of expert systems there appeared the need to elaborate the method of formalization the medical diagnosis making. This can be profitable not only for expert systems; nowadays when the amount of medical data is big and still increasing, there is the need to make some order in the chaos of information. Formalization of inexact methods of reasoning in medicine seems to be urgent.

In this paper we present the proposition of the application of the Dempster's rule in medical diagnosis making.

The paper is organized as follows: In Section 2nd mathematical background is presented. Section 3 is devoted to problems of reasoning with uncertain information, and Section 4 provides examples from the field of medical diagnosis.

2. Mathematical background

2.1 Basic notions of random set theory

Let $(Y, 2^Y, P)$ be a probability space and K be a multivaled mapping from Y into a nonvoid space X,

$$K: Y \to 2^X.$$

For practical applications it sufficies to assume that X as well as Y are finite sets.

Viewing the power set 2^X as a set of „elementary events" we can build a new space of „composite events" denoted 4^X for simplicity (4^X is the power set of 2^X). Proceeding in a standard way we get a probability measure Pr on 4^X induced through K, namely

$$Pr(F)=P(\{y \in Y/K(y) \in F\}), \quad F \in 4^X \tag{1}$$

The triple $(2^X, 4^X, Pr)$ is referred to as the canonical random set [3]. The probability measure Pr can be determined in terms of a much simpler set function defined as

$$T(A)=\text{Prob}(K \cap A \neq \emptyset), \quad A \in 2^X, \tag{2}$$

determining the probability that the random set K hits a given subset A of X. The functional T is a counterpart of the distribution function. To show that this is the case we proceed along the next steps being the corollaries of Lemmas 2.2.1 and 2.2.2 in [3]. (For rigorous proofs of these steps an interested reader is referred to [8]).

First we introduce the family $\mathcal{L}$ containing the classes of the form

$$L=A_0^E \bigcap_{i=1}^{k} A_i^N, \quad A_0, A_1, \ldots, A_k \in X, \tag{3}$$

where

$$A^E=\{M \in 2^X : A \cap M=\emptyset\}, \tag{4a}$$

$$A^N=2^X \backslash A^E \tag{4b}$$

and $k \leqslant n$, $n=\text{card}(X)$. We verity that each L can be written in the reduced form $L=A_0^E \bigcap_{i=1}^{k} A_i^N$ where (a) $A_0 \cap A_i=\emptyset$ and (b) $A_i \not\subseteq A_j, j \neq i$. Morever $L=\emptyset$ if and only if there exists A_i such that $A_i \subseteq A_0$.

Using these properties we state that $\mathcal{L}$ is a semialgebra. In third step we make use of Choquet's capacity of order n, i.e. of a set function T defined on 2^X such that (in the discrete case)

(C1) $T(A) \leqslant T(B)$ if $A \subset B$, $A, B \in 2^X$,

(C2) for any sequence $A_0, A_1, \ldots, A_k$, $k \leqslant n$, of subsets of X the successive differences

$$S_0(A_0)=1-T(A_0)$$

$$S_k(A_0; A_1 \ldots A_k)=S_{k-1}(A_0; A_1 \ldots A_{k-1})-S_{k-1}(A_0 \cup A_k; A_1 \ldots A_{k-1})$$

are nonnegative. We prove that if T is a Chquet capacity of order n such that $T(\emptyset)=0$, $0 \leqslant T \leqslant 1$ then there exists an additive mapping $P': \mathcal{L} \to [0, 1]$ defined as

$$P'(L)=S_k(A_0; A_1 \ldots A_k), \tag{5}$$

and such that $P'(\emptyset)=0$. In particular $P'(A^N)=T(A)$.

Finally, using standard results of measure theory, cf. [4], we extend this P' to a probability measure Pr on 4^X. This is the theme of Choquet's theorem: Let X be a discrete space and T be a Choquet capacity of order 2^n defined on 2^X and such that $T(\emptyset)=0$, $0 \leqslant T \leqslant 1$.

Then the assignment.

$$Pr(A^N)=T(A), \quad A \in 2^X, \tag{6}$$

induces a unique probability measure over 4^X.

2.2 Relations to the Dempster-Shafer theory

Noticing that

$$Pr\left(A_0^E \bigcap_{i=1}^{k} A_i^N\right)=\text{Prob}(K\cap A_o=\emptyset \;\&\; K\cap A_1\neq\emptyset \;\&\ldots\&\; K\cap A_k\neq\emptyset) \tag{7}$$

we obtain the next expressions

$$\text{Prob}(K\cap A\neq \emptyset)=\text{Pr}(A^N), \tag{8}$$

$$\text{Prob}(K\subseteq A)=\text{Pr}(\bar{A}^E), \tag{9}$$

$$\text{Prob}(K\supseteq A)=\text{Pr}\Big(\bigcap_{x\in A}\{x\}^N\Big), \tag{10}$$

$$\text{Prob}(K=A)=\text{Pr}\Big(\bar{A}^E \bigcap_{x\in A}\{x\}^N\Big), \tag{11}$$

where $\bar{A}$ stands for the complement of A in X.

For example the equation (9) means that the probability that the random set K is contained in $A\subseteq X$ equals to $\text{Pr}(\bar{A}^E)$, or noticing that

$$\text{Pr}(\bar{A}^E)=1-\text{Pr}(A^N) \tag{12}$$

we state that

$$\text{Prob}(K\subseteq A)=1-(\text{Prob}(K\cap\bar{A}\neq\emptyset)). \tag{13}$$

Consider now the set functions f_1, f_2 defined as the conditional probabilities

$$f_1(A)=\text{Prob}(K\subseteq A/K\neq\emptyset)=\text{Pr}(\bar{A}^E\cap X^N)/\text{Pr}(X^N), \tag{14}$$

$$f_2(A)=\text{Prob}(K\cap A\neq\emptyset/K\neq\emptyset)=\text{Pr}(A^N\cap X^N)/\text{Pr}(X^N). \tag{15}$$

We easily verify (see [8] for a rigorous proof) that

(B1) $f_1(\emptyset)=\emptyset$,

(B2) $f_1(X)=1$,

(B3) for any sequence $A_1 \ldots A_k$ of subsets of X we have

$$f_1\left(\bigcup_{i=1}^{k} A_i\right)\geqslant \sum_{\substack{I\in\{1\ldots k\}\\ I\neq\emptyset}} (-1)^{|I|+1} f_1\Big(\bigcap_{i\in I} A_i\Big).$$

Here $|I|$ stands for the cardinality of the set I. From these properties it follows that the set function (14) qualifies as Shafer's belief function [6]. We will write Bel instead of f_1. Similarly f_2 corresponds to Shafer's plausibility functions Pl.

Noticing that

$$\bar{A}^E=\{M \in 2^X | M \subseteq A\} \tag{16}$$

and using the additivity property of probability measure we get another characterization of belief function, namely

$$\mathrm{Bel}(A)=\sum_{M \subseteq A} m(M), \tag{17}$$

where

$$m(M)=\mathrm{Prob}(K=M)/\mathrm{Pr}(X^N), \tag{18}$$

is Shafer's basic probability assignment function.

Similarly the set function

$$Q(A)=\mathrm{Prob}(K \supseteq A / K \neq \emptyset) \tag{19}$$

agrees with the commonality function.

Applying the recurrent definition (C1) and (C2) (not the Möbius inversion theorem!) we recover further relationships among the set functions just introduced. For example

$$m(M)=\sum_{A \subseteq M} (-1)^{|M|-|A|} \mathrm{Bel}(A), \tag{20}$$

$$m(M)=\sum_{A \supseteq M} (-1)^{|A|-|M|} Q(A). \tag{21}$$

Suppose now K_1, K_2 are two independent random sets, i.e.

$\mathrm{Prob}(K_1=M_1,\ K_2=M_2)=\mathrm{Prob}(K_1=M_1) \cdot \mathrm{Prob}(K_2=M_2)$ and consider the random set being the set-theoretical intersection of K_1 and K_2.

Proceeding in a standard way we state that

$$\mathrm{Prob}(K=M)=\sum_{\substack{M_1, M_2 \\ M_1 \cap M_2 = M}} \mathrm{Prob}(K_1=M_1) \cdot \mathrm{Prob}(K_2=M_2). \tag{22}$$

A particular example of (22) is the Dempster's rule of combination. As in the Dempster-Shafer theory we deal with belief functions only this rule takes the form

$$m(M)=\mathrm{Prob}(K_1 \cap K_2 = M | K_1 \cap K_2 \neq \emptyset)=$$

$$=\frac{\sum\{m_1(M_1) m_2(M_2)/M_1,\ M_2 \in X,\ M_1 \cap M_2 = M\}}{\sum\{m_1(N_1) m_2(N_2)/N_1,\ N_2 \in X,\ N_1 \cap N_2 \neq \emptyset\}}. \tag{23}$$

The denominator d of (23) can be viewed a a measure of conflict between the combined belief functions. When $d=1$ there is no conflict and when $d=0$ we are faced with a contradictory evidence.

3. Inferences with uncertain propositions

Suppose now X is a finite set of atomic propositions and let us identify 2^X with the set of all composite propositions derived from X by using the logical connectives $\neg$, $\vee$, $\wedge$, $\rightarrow$ (negation, disjunction, conjunction and implication respectively). We will say that a proposition p is uncertain if its truth (or its falsity) cannot be definitely established [5].

Recalling the definition of canonical random set we can imagine the impact of evidence as the assignment of different portion of belief to propositions from 2^X.

In other words the evidence may support some propositions from 2^X and this is expressed numerically in terms of the basic probability assignment, m. Now, the total portion of belief commited to a proportion p equals to

$$\mathrm{Bel}(p)=\sum_{\substack{q\in 2^X\\ q\rightarrow p}} m(q) \tag{24}$$

and

$$\mathrm{Pl}(p)=\sum_{\substack{q\in 2^X\\ \neg(q\rightarrow p)}} m(q). \tag{25}$$

One should notice that

$$\mathrm{Bel}(p)+\mathrm{Bel}(\neg p)\leqslant 1, \tag{26}$$

$$\mathrm{Pl}(p)+\mathrm{Pl}(\neg p)\geqslant 1, \tag{27}$$

$$\mathrm{Bel}(p)+\mathrm{Pl}(\neg p)=1. \tag{28}$$

With this approach it is possible to distinguish between the lack of belief and disbelief. Indeed, since $\mathrm{Bel}(p)\leqslant\mathrm{Pl}(p)$ we state that

(a) $\mathrm{Bel}(p)=0$ does not entail $\mathrm{Bel}(\neg p)=1$,
(b) $\mathrm{Bel}(\neg p)=1$ entails $\mathrm{Bel}(p)=0$.

It is convenient to represent an uncertain proposition p by a pair of numbers $(\mathrm{Bel}(p), \mathrm{Pl}(p))$. The number

$$\mathrm{I}(p)=\mathrm{Pl}(p)-\mathrm{Bel}(p), \tag{29}$$

is said to be the width of a belief interval. It measures the amount of uncertainty with respect to the truth of our hypothesis. Suppose now we want to infer about q with the rule „If p then q". Treating this rule as a logical implication we can write

$$\begin{aligned}\mathrm{Bel}(q)&=\mathrm{Bel}[(q\vee p)\wedge(\neg p\vee q)]\geqslant\\ &\geqslant\mathrm{Bel}(q\vee p)+\mathrm{Bel}(\neg p\vee q)-\mathrm{Bel}[(q\vee p)\vee(q\vee\neg p)]=\\ &=\mathrm{Bel}(q\vee p)+\mathrm{Bel}(\neg p\vee q)-1\geqslant\\ &\geqslant\mathrm{Bel}(p)+\mathrm{Bel}(\neg p\vee q)-1.\end{aligned} \tag{30}$$

That is we get the next pattern of inference (Modus Ponens)

$$\begin{array}{l} \mathrm{Bel}(p) \geqslant a \\ \mathrm{Bel}(p \rightarrow q) \geqslant b \\ \hline \mathrm{Bel}(q) \geqslant \max(0,\ a+b-1) \end{array} \tag{31}$$

being a counterpart of Łukasiewicz's implication. (See [5] for an exhaustive discussion). However, the lower bound in (31) seems to be too rough estimate for $\mathrm{Bel}(q)$.

To get more precise value of $\mathrm{Bel}(q)$ we employ the idea proposed in [1]

Consider again the rule: „If p then q". This rule generates its own frame of discerment constructed from four propositions: p, $\neg p$, q, $\neg q$. The elements of this frame are subsets of the set

$$X = \{\langle p, q\rangle, \langle \neg p, q\rangle, \langle \neg p, \neg q\rangle, \langle p, \neg q\rangle, \langle p, \neg q\rangle\}. \tag{32}$$

For example

$\langle p, q\rangle$ – means p is true and q is true

$(\langle p, q\rangle, \langle p, \neg q\rangle) \equiv (p, *\rangle$ – means p is true and so on.

This approach offers flexibility in determining the context of a rule.

To demonstrate it assume that we decided to treat the rule as a logical implication. Since $p \rightarrow q$ is false only if p is true and q is false we employ the next basic probability assignment m_R

$$m_R(r) = \begin{cases} \alpha \text{ when } r \equiv \{\langle p \vee q\rangle \langle \neg p \vee q\rangle \langle \neg p \vee \neg q\rangle\}, \\ 1-\alpha \text{ when } r \equiv X \\ 0 \text{ otherwise}, \end{cases}$$

where $0 < \alpha \leqslant 1$. To express our belief on the premise p let us use the m – function

$$m_P(r) = \begin{cases} \beta \text{ when } r \equiv \langle p * \rangle, \\ \gamma \text{ when } r \equiv \langle \neg p * \rangle, \\ 1-\beta-\gamma \text{ when } r \equiv X, \\ 0 \text{ otherwise}. \end{cases}$$

Applying the Dempster rule of combination (23) we get the next basic probability assignment function m

$$m(r) = \begin{cases} \alpha,\ \beta & \text{when } r \equiv \langle p, q\rangle, \\ \beta(1-\alpha) & \text{when } r \equiv \langle p * \rangle, \\ \gamma & \text{when } r \equiv \langle \neg p * \rangle, \\ \alpha(1-\beta-\gamma) & \text{when } r \equiv \{\langle pq\rangle, \langle \neg pq\rangle, \langle \neg p \neg q\rangle\}, \\ (1-\alpha)(1-\beta-\gamma) & \text{when } r \equiv X, \\ 0 & \text{otherwise}. \end{cases}$$

Restricting resulting belief function to the function defined over the frame 2^Q, where $Q \equiv (q, \neg q)$ – see [6] for details – we obtain $\mathrm{Bel}(q) = \alpha\beta$, $\mathrm{Bel}(\neg q) = 0$. In other words we get the next pattern of reasoning

$$\begin{array}{l} \mathrm{Bel}(p) = a \\ \mathrm{Bel}(p \to q) = b \\ \hline \mathrm{Bel}(q) = ab \end{array} \tag{33}$$

compatible with (31)

In summary the procedure described above consists of the following steps

1) Both user and expert define its own belief function corresponding to a rule

2) The two belief functions are combined by the Dempster rule of combination

(3) The resulting belief function is then restricted to a frame containing the meaningful propositions.

4. Applications to medical diagnosis

Typically, medical knowledge is represented in the form of IF – THEN statements, where IF part describes symptoms or observations and consequent refers to a diagnosis or judgement. Being „rules-of-thumb" these statements can be viewed as the uncertain propositions. The same applies to symptoms, as each symptom is a label describing an abstract health status of a patient. Observing a real patient we only suppose that his state can be labelled by one of earlier defined abstract terms (symptoms).

Thus Bel (s) – where s stands for a symptom – expresses physician's degree of belief that the health status of a patient can be described by s. The problem of determining a degree of belief which can be attached to a rule is more complicated. Suppose that our rule is of the form „If s then d". Exploiting the idea described in Section 3 we build a frame of discernment associated to the rule. This frame is a subset of the power set of

$$X = \{\langle \neg s, \neg d\rangle, \langle s, d\rangle, \langle s, \neg d\rangle, \langle \neg s, d\rangle\}$$

and it contains all relevant or meaningful statements which can be derived from the set X of elementary propositions. For instance our rule can be interpreted in the next competiting ways

1) If symptom s has been observed then the judgement d is appropriate (to the degree t_1).

2) If symptom s has not been observed then the judgement d is appropriate (to the degree t_2).

3) The judgement d is appropriate to the degree t_3 (n_0 matter s has or has not been observed).

The three interpretations meet now on the common groud, that is on the frame of discernment. We represent them by using the basic probability assignment function, that is

$$m(\langle s, d\rangle)=t_1,$$
$$m(\langle \neg s, d\rangle)=t_2,$$
$$m(\langle s, d\rangle, \langle \neg s, d\rangle)=t_3,$$

provided that $t_1+t_2+t_3=1$. When this is not the case we should add one more interpretation

$$m(X)=1-t_1-t_2-t_3,$$

representing our ignorance about the judgement d.

To be able to combine the two evidences (first provided by a general knowledge and second provided by actual observation) we extend Bel (s) to the frame just introduced, e.g.

$$m(\langle s, *\rangle)=w_1,$$
$$m(\langle \neg s, *\rangle)=w_2,$$
$$m(X)=1-w_1-w_2.$$

Now, combining two belief functions over 2^X we get the resulting belief function that supports our belief concerning d itself. This belief function is obtained by restricting Bel over 2^X to Bel' over 2^D, where $D=\{d, \neg d\}$.

To be more illustrative let us consider the problem of a severely burnt patient. One of the symptoms which should be quickly recognised is the imparment of the respiratory tract. Early recognition of this complication allows a physician to intubate the patient before the swelling makes it impossible. Two examples listed below demonstrate how the diagnostic process can be automatized.

Example 1. Rule: If cyanosis is present then there is some weak evidence that the respiratory tract is impaired.

Let us denote cyanosis by c and the impairment of the respiratory tract by i. The set of possibilities is the following:

$$X=\{\langle c, i\rangle, \langle \neg c, i\rangle, \langle c, \neg i\rangle, \langle \neg c, \neg i\rangle\}.$$

The total number of all possible subsets is 16 (2^4) but not all of them are of diagnostic interest.

The quantity m (A) is a measure of belief commited exactly to A. The total belief is 1.

According to an expert opinion, the rule can be quantified in the next way:

$$m_1(\langle c, i\rangle)=0.3, \quad m_1(\langle \neg c, i\rangle), \quad m_1(\langle c, \neg i\rangle)=0.1,$$
$$m_1(\langle \neg c, \neg i\rangle)=0.2, \quad m_1(X)=0.2,$$

where X represents all possibilities.

Let us imagine that the physician determines that the cyanosis is present. His measure of belief of the symptom is 0.8

$$m_{s1}(\langle c, i\rangle\ \langle c, \neg i\rangle)=0.8$$

The combination of two belief functions can be represented graphically as follows

X 0.2		
$\langle c, i\rangle$ 0.3		
$\langle \neg c, i\rangle$ 0.2	Ø	
$\langle c, \neg i\rangle$ 0.1		
$\langle \neg c, \neg i\rangle$ 0.2	Ø	
	$\langle c, i\rangle, \langle c, \neg i\rangle$ 0.8	X 0.2

The degree of conflict equals to

$$k=1-0.8\times 0.2+0.8\times 0.2=0.68$$

and the resulting basic probability assignments are

$$m(\langle c, \neg i\rangle)=0.08+0.02=0.1,\quad m(\langle c, i\rangle)=0.24+0.06=0.3,$$

$$m(\langle c, i\rangle, \langle c, \neg i\rangle)=0.16,\quad m(\langle \neg c, i\rangle)=0.04,\quad m(\langle \neg c, \neg i\rangle)=0.04.$$

Thus our belief function over 2^X is

$$\mathrm{Bel}(\langle c, \neg i\rangle)=10/88,\quad \mathrm{Bel}(\langle c, i\rangle)=30/68,\quad \mathrm{Bel}(\langle \neg c, i\rangle)=4/68,$$

$$\mathrm{Bel}(\langle \neg c, \neg i\rangle)=4/68,\quad \mathrm{Bel}(\langle c, i\rangle, \langle c, \neg i\rangle)=16/68,\quad \mathrm{Bel}(X)=4/68.$$

Restricting this Bel to 2^I, where $I=\{i, \neg i\}$ we state that the measure of belief of the impairment of the respiratory tract

$$\mathrm{Bel}(i)=\mathrm{Bel}(\langle c, i\rangle)+\mathrm{Bel}(\langle \neg c, i\rangle)=34/68=0.5$$

and

$$\mathrm{Bel}(\neg i)=\mathrm{Bel}(\langle c, \neg i\rangle)+\mathrm{Bel}(\langle \neg c, \neg i\rangle)=14/68=0.2$$

The problem is more interesting when there are several symptoms suggesting the impairment of the respiratory tract.

Example 2. Let us consider the following case:

The patient's face is burnt – the measure of belief=1.0

There are blisters round his mouth : the measure of belief=0.9

His voice is hoarse – the measure of belief=0.6

The following rules should be considered:

1) If the face is burnt then there exists some evidence that the respiratory tract is impaired Let us denote the burns of the faced by f. According to an

expert's opinion:

$$m_2(\langle f, i\rangle)=0.4, \quad m_2(\langle f, \neg i\rangle)=0.2, \quad m_2(\langle \neg f, i\rangle)=0.1,$$

$$m_2(\langle \neg f, \neg i\rangle)=0.2, \quad m_2(X)=0.1.$$

2) If the blisters are present in the mouth or round the mouth then there exists some evidence that the respiratory tract is impaired. Let us denote the blisters by b. According to an expert's opinion:

$$m_3(\langle b, i\rangle)=0.5, \quad m_3(\langle b, \neg i\rangle)=0.1, \quad m_3(\langle \neg b, i\rangle)=0.3, \quad m_3(X)=0.1.$$

3) If the patient's voice is hoarse then there exists some evidence that the respiratory tract is impaired. Let us denote the hoarse throat by h. According to an expert's opinion:

$$m_4(\langle h, i\rangle)=0.6, \quad m_4(\langle h, \neg i\rangle)=0.1, \quad m_4(\langle \neg h, i\rangle)=0.1, \quad m_4(X)=0.2.$$

As we use now three rules the process of combination is more complicated. First we compute the degree Bel (i) using the first rule only.

X 0.1		
$\langle \neg f, \neg i\rangle$ 0.2	Ø	
$\langle \neg f, i\rangle$ 0.1	Ø	
$\langle f, \neg i\rangle$ 0.2		
$\langle f, i\rangle$ 0.4		
	$\langle f, i\rangle, \langle f, \neg i\rangle$ 1.0	X 0

We get the next results

$$k=1-0.3=0.7,$$

$$m(\langle f, i\rangle)=0.4, \quad m(\langle f, \neg i\rangle)=0.2, \quad m(\langle \neg f, i\rangle)=0,$$

$$m(\langle \neg f, \neg i\rangle)=0, \quad m(\langle f, i\rangle, \langle f, \neg i\rangle)=0.1, \quad m(X)=0.$$

Thus our belief that the intubation is needed equals to

$$\mathrm{Bel}_1(i)=4/7, \quad \mathrm{Bel}_1(\neg i)=2/7, \quad \mathrm{Bel}_1(X)=1/7.$$

when the first symptom is considered.

Combining the belief function corresponding to the second rule with our belief concerning the presence of blisters around patient's mouth we get

X 0.1		
$\langle \neg b, i\rangle$ 0.3	Ø	
$\langle b, \neg i\rangle$ 0.1		
$\langle b, i\rangle$ 0.5		
	$\langle b, i\rangle, \langle b, \neg i\rangle$ 0.9	X 0.1

$$k = 1 - 0.3 \times 0.9 = 0.73,$$

$$m(\langle b, i\rangle) = 0.5, \quad m(\langle b, \neg i\rangle) = 0.1, \quad m(\langle \neg b, i\rangle) = 0.03,$$

$$m(\langle b, i\rangle, \langle b, \neg i\rangle) = 0.09, \quad m(X) = 0.01.$$

The need for intubation – based on second symptom only – is quantified as

$$\mathrm{Bel}_2(i) = 53/73, \quad \mathrm{Bel}_2(\neg i) = 10/73, \quad \mathrm{Bel}_2(X) = 10/73.$$

Finally, exploiting the third rule we obtain

X 0.2		
$\langle \neg h, i\rangle$ 0.1	Ø	
$\langle h, \neg i\rangle$ 0.1		
$\langle h, i\rangle$ 0.6		
	$\langle h, i\rangle, \langle h, \neg i\rangle$ 0.6	X 0.4

$$k = 1 - 0.06 = 0.94,$$

$$m\{\langle h, i\rangle\} = 0.6, \quad m(\langle h, \neg i\rangle) = 0.1, \quad m(\langle \neg h, i\rangle) = 0.04,$$

$$m(\langle h, i\rangle, \langle h, \neg i\rangle) = 0.12, \quad m(X) = 0.08.$$

$$\mathrm{Bel}_3(i) = 64/94, \quad \mathrm{Bel}_3(\neg i) = 10/94, \quad \mathrm{Bel}_3(X) = 20/94.$$

To find the resulting belief concerning the need of intubation we combine the three belief functions obtaining

$$\mathrm{Bel}_{1,2}(i) = \mathrm{Bel}_1 \oplus \mathrm{Bel}_2(i) = 305/365,$$

$$\mathrm{Bel}_{1,2}(\neg i) = \mathrm{Bel}_1 \oplus \mathrm{Bel}_2(\neg i) = 50/365),$$

$$\mathrm{Bel}_{1,2}\, X = \mathrm{Bel}_1 \oplus \mathrm{Bel}_2(X) = 10/365,$$

$$\mathrm{Bel}(i) = \mathrm{Bel}_{1,2} \oplus \mathrm{Bel}_3(i) = 0.935,$$

$$\mathrm{Bel}(\neg i) = \mathrm{Bel}_{1,2} \oplus \mathrm{Bel}_3(\neg i) = 0.057.$$

Computing the degress of plausibility

$$\mathrm{Pl}(i)=0.943, \quad \mathrm{Pl}(\neg i)=0.065$$

as well as the width of belief intervals

$$I(i)=I(\neg i)=0.008$$

we state that the symptoms confirm the nead of intubation in a high degree and almost precise.

It is noticeable that our final result is much stroger in comparison with the degrees offered by the rules considered separately.

5. Concluding remarks

The results obtained in the quoted examples are quite satisfactory from the medical stainpoint. This gives us an impugn to build a small expert system in the field of fast medical decision – making.

One should notice, however, that our examples concern a simply structured problem. We used rules without chaining them. This task will be elaborated elsewhere.

References

[1] W. F. Eddy, G. P. Pei, Structures of rule based belief function, IBM J. of Res. & Developement, 30 (1986) No 1, 93-101.

[2] J. Gordon, E. H. Shortliffe A Method for Managing Evidential Reasoning in a hierarchical Hypothesis Space, AI 26 (1985) 323-357.

[3] G. Matheron, Random Sets and Integral Geometry, Willey 1975.

[4] K. R. Parthasharathy, Introduction to Probability & Measure, Willey 1980.

[5] H. Prade, A computational approach to approximate and plausible reasoning with applications to expert systems, IEEE Trans. Pattern Analysis and Machine Intelligence, vol PAMI – 7 (1985) 260-283.

[6] G. Shafer, A Mathematical Theory of Evidence, Princeton, 1976.

[7] S. T. Wierzchoń, On plausible reasoning, to appear in "Fuzzy Computing" (M. M. Gupta, ed) North Holland 1988.

[8] S. T. Wierzchoń, Random Sets – Basis for belief and possibility structures, in preparation.

BIBLIOTEKA AE w Poznaniu